MR3408454 (Review)   13-01 03F65 13C10

Lombardi, Henri (F-FRAN-NDM); **Quitté, Claude** (F-POIT-NDM)

★**Commutative algebra: constructive methods.**
Finite projective modules.
Updated and revised edition.
Translated from the French by Tania K. Roblot.
Algebra and Applications, 20.
Springer, *Dordrecht*, 2015. *xlix+996 pp. ISBN* 978-94-017-9943-0; 978-94-017-9944-7

The book under review, as the title says, deals with commutative algebra from a constructive point of view. The authors would term the reviewer a 'classical mathematician'.

The book treats many of the standard topics from commutative algebra and projective modules in great detail, all using constructive methods. For example, localization, Galois theory, Krull dimension, integral extensions, flatness, the Forster-Swan Theorem, Bass-Serre Theorems, and the Quillen-Suslin Theorem, among many others, are covered in the book. The exhaustive bibliography draws both from classical and constructive mathematics.

The overarching themes in the book are various local-global principles and certain dynamic methods. These help both at theoretical and practical levels. Remember that constructive methods in general do not admit results like Zorn's lemma, or even more seriously, the law of the excluded middle. So, from the classical point of view, the proofs are complicated, but necessary for constructive arguments.

Some of the statements can be a bit misleading. For example, lack of an algebraic proof that the real sphere cannot be combed (page 7) is not quite true; with the advent of Euler classes (and more generally, resorting to Chow groups) there are algebraic proofs of existence of stably free non-free modules. Similarly, the authors state some of the known cases of the Bass-Quillen conjecture (page 923). But they omit some of the most general cases due to H. Lindel and others. It is not clear to me whether the authors are merely saying that no constructive methods are known to establish these. Another minor problem is that the English translation from the French is a bit rough (not surprising for such a long book), but should cause no mathematical difficulty.

This will be a great addition to the bookshelves of the community of constructive mathematicians.   *N. Mohan Kumar*